A minimum spanning tree (MST) or minimum weight spanning tree is a subset of the edges of a connected, edge-weighted undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight. That is, it is a spanning tree whose sum of edge weights is as small as possible. More generally, any edge-weighted undirected graph (not necessarily connected) has a minimum spanning forest, which is a union of the minimum spanning trees for its connected components (<https://en.wikipedia.org/wiki/Minimum_spanning_tree>). Consider the graph:

Diagram, schematic

Description automatically generated

There are two possible minimum spanning trees:

Diagram, schematic

Description automatically generated and A picture containing text, clock

Description automatically generatedIn each case, all vertices are in the path, and the path has lowest edge weight (whatever metric that might be).

This is determined by following all possible spanning paths and totaling their weight. Then listing the paths that all have the minimum weight. Here is an exhaustive list of paths and their weights:

Table

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Green represents the lowest paths between the same vertices. Yellow means either path works.